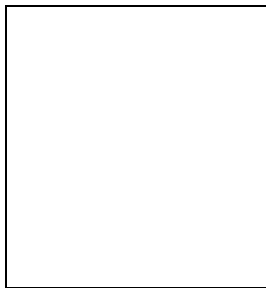


Phenomenology of next-leading BFKL

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We propose a phenomenological study of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach applied to the data on the proton structure function F_2 measured at HERA in the small- x_{Bj} region ($x_{Bj} < 0.01$) and Q^2 in the range 5 to 120 GeV². With a simplified “effective kernel” approximation, we present a comparison between leading-logs (LO) and next-to-leading logs (NLO) BFKL approaches in the saddle-point approximation, using known resummed NLO-BFKL kernels. The LO result gives a very good description of the data with a three parameters fit of F_2 but an unphysical value of the strong coupling constant, whereas the NLO two parameters fit leads to a qualitatively satisfactory account of the running coupling constant effect but quantitatively, for $Q^2 < 10$ GeV², it fails to reproduce properly the data.

1 Introduction

Precise measurements of the proton structure function F_2 at small x_{Bj} ($x_{Bj} < 0.01$)¹ have led to important tests of QCD evolution equations and a better understanding of deep inelastic scattering phenomenology. For the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi² (DGLAP) evolution in Q^2 , it has been possible to test it in various ways with NLO (next-to-leading log Q^2) and now NNLO accuracy and it works quite well in a large range of Q^2 and x_{Bj} . Testing precisely the Balitsky-Fadin-Kuraev-Lipatov³ (BFKL) evolution in x_{Bj} beyond leading order appears more difficult and it is the aim of the following analysis.

In a first part, we perform a LO-BFKL analysis of F_2 measurements¹, reproducing results obtained previously with older data⁴. However, as it was found in reference⁴, we have to introduce an effective but unphysical value of the strong coupling constant $\alpha \sim 0.1$ instead of $\alpha \sim 0.2$ predicted for the Q^2 -range considered here. It reveals the need for NLO corrections.

In fact, the theoretical task of computing these corrections appears to be quite hard. It is now in good progress but still under completion for the coupling to external particles. For the BFKL kernel, they have been calculated after much efforts⁵. In addition, for the whole theoretical

approach to be correct, an appropriate resummation of spurious singularities, brought together with the NLO corrections, has to be performed at all orders of the perturbative expansion⁶. This resummation procedure is required by consistency with the QCD renormalization group. Various resummation schemes have been proposed^{6,7,8} which satisfy the renormalization group requirements while retaining the computed value of the NLO terms in the BFKL kernel. In the following, we present an NLO-BFKL analysis for the scheme labeled *S3* in reference⁶. A more complete treatment can be found in reference⁹, where we compare the different NLO schemes in order to distinguish between different resummation options.

2 “Effective kernel” and saddle-point approximation of BFKL amplitudes

The BFKL formulation of the proton structure functions can be formulated in terms of the double inverse Mellin integral

$$F_2 = \int \int \frac{d\gamma d\omega}{(2i\pi)^2} \left(\frac{Q^2}{Q_0^2} \right)^\gamma x_{Bj}^{-\omega} \mathcal{F}_2(\gamma, \omega) . \quad (1)$$

At LO level one has (see, e.g.⁴)

$$\mathcal{F}_2(\gamma, \omega) = \frac{h_2(\gamma, \omega)}{\omega - \bar{\alpha} \chi_{L0}(\gamma)} \quad (2)$$

where $\bar{\alpha} \equiv \alpha_s N_c / \pi$, α_s is the coupling constant which is merely a parameter at this LO level. The LO BFKL kernel is written as

$$\chi_{L0}(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma) . \quad (3)$$

$h_2(\gamma, \omega)$ is a prefactor which takes into account both the phenomenological non-perturbative coupling to the proton and the perturbative coupling to the virtual photon. Note that the variable γ plays the role of a continuous anomalous dimension while ω is the continuous index of the Mellin moment conjugated with the rapidity $Y \equiv \log 1/x_{Bj}$.

Recalling well-known properties of LO-BFKL amplitudes, one assumes that $h_2(\gamma, \omega)$ is regular. The pole contribution at $\omega = \bar{\alpha} \chi_{L0}(\gamma)$ in (2) leads to a single Mellin transform in γ for which one may use a saddle-point approximation at small values of x_{Bj} , which is known to give a very good account of the phenomenology at LO

$$F_2(x, Q^2) \approx \mathcal{N} \exp \left\{ \frac{L}{2} + \alpha_s Y \chi_{L0}(\tfrac{1}{2}) - \frac{L^2}{2\alpha_s Y \chi_{L0}''(\tfrac{1}{2})} \right\} , \quad (4)$$

where $L \equiv \log(Q^2/Q_0^2)$ and \mathcal{N} is a normalisation taking into account all the smooth prefactors^a. As a consequence, the only three relevant parameters in (4) are \mathcal{N} , $\bar{\alpha}$ and Q_0 . In this picture $\bar{\alpha}$ has to be considered as a parameter and not a genuine QCD coupling constant since the value obtained in the fits is not related to the coupling constant values in the considered range of Q^2 . The result of the fit is presented on figure 2 together with the measurements of F_2 by the H1 collaboration¹. We obtain a very good agreement (at the χ^2 level of the NLO DGLAP fit) between this simple parameterisation and F_2 data.

The treatment of the NLO BFKL kernel (expressed with the resummation scheme *S3*) is detailed in reference⁹. It retains the running property of the QCD coupling constant with its theoretically predetermined value at the relevant Q^2 range. It is shown that an “effective kernel”

^aIn particular, the square root prefactor of the gaussian saddle-point approximation can be merged in the normalization.

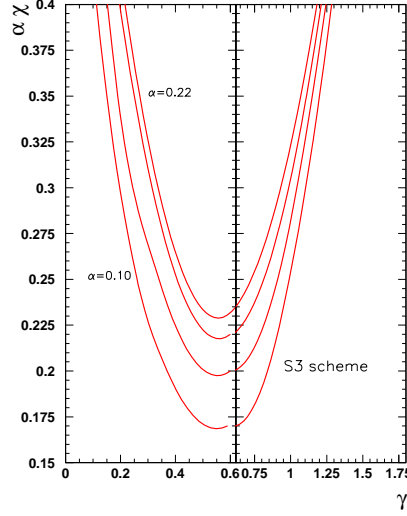


Figure 1: $\alpha_{RG}\chi$ as a function of γ for different values of α_{RG} for $S3$ scheme (α_{RG} varies between 0.1 to 0.24).

approximation associated with a saddle-point expression similar to (4) allows one to obtain a simple formula for the structure function F_2

$$F_2(x, Q^2) \approx \mathcal{N} \exp \left\{ \gamma_c L + \alpha_{RG} \chi_{eff}(\gamma_c, \alpha_{RG}) Y - \frac{L^2}{2\alpha_{RG}\chi''_{eff}(\gamma_c, \alpha_{RG}) Y} \right\}, \quad (5)$$

where χ_{eff} is the "effective kernel" at NLO which is shown in figure 1 and γ_c is defined by the implicit saddle-point equation

$$\frac{\partial \chi_{eff}}{\partial \gamma}(\gamma_c, \alpha_{RG}(Q^2)) = 0 \quad \left[\alpha_{RG}(Q^2) \right]^{-1} \equiv b \log \left(Q^2 / \Lambda_{QCD}^2 \right). \quad (6)$$

with $b = 11/12 - 1/6 N_f/N_c$. It is important at this stage to notice that the formula (5) has only two free parameters \mathcal{N} and Q_0 instead of three for the LO case (4), as we are using the QCD universal coupling constant α_{RG} . It allows one to compare in a similar footing the LO and NLO BFKL kernels to F_2 data, as presented on figure 2. A global good agreement is obtained but the for $Q^2 < 10 \text{ GeV}^2$, we notice that the NLO BFKL fit fails to reproduce properly the F_2 data. A similar conclusion holds for other available resummation schemes⁹.

3 Conclusion

We have confronted the predictions of BFKL kernels at the level of leading and next-leading logarithms (scheme $S3$) with structure function data. We have proposed to use the "effective kernel" approximation of the NLO-BFKL kernels which, associated with the usual saddle-point approximation at high rapidity and large enough Q^2 , allows one to obtain a simple two parameters formula for the structure function F_2 . The comparison with the similar three parameters formula commonly used at LO level shows a deterioration of the fits for $Q^2 < 10 \text{ GeV}^2$. These deviations are under study and could originate from different sources as the non validity of the saddle-point approximation at low Q^2 or unknown aspects of the prefactors, in particular the non-perturbative ones.

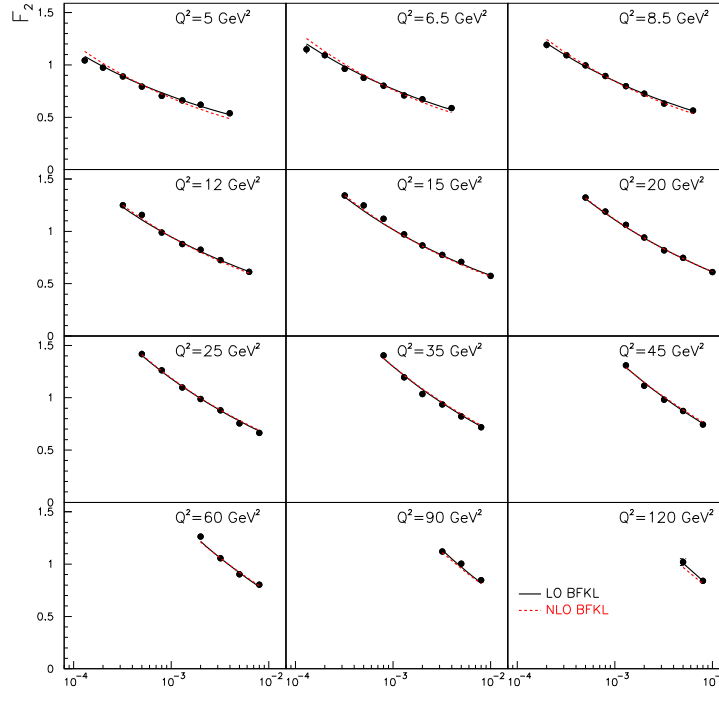


Figure 2: Results of the BFKL fits to the H1 data: LO (continuous lines) and NLO -scheme $S3$ - (dashed lines).

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